Rational Zero Theorem

What do we do if we're not told one of the factors?

Every rational zero has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term}}{\text{factor of leading coefficient}}$$

Example: List possible rational zeros:

$f(x) = 2x^3 + x^2 - 3x - 6$

constant: (p) = -6 factors: $\pm1, \pm2, \pm3, \pm6$

lead coefficient: (q) = 2 factors: $\pm1, \pm2$

Possible rational zeros: $\frac{p}{q} = \frac{\pm1}{\pm1}, \frac{\pm2}{\pm1}, \frac{\pm3}{\pm1}, \frac{\pm6}{\pm1},$

$\frac{1}{1}\ \frac{1}{-1}\ \frac{-1}{1}\ \frac{-1}{-1}$

$1\ \ -1\ \ -1\ \ 1$

$\frac{\pm1}{\pm2}, \frac{\pm2}{\pm2}, \frac{\pm3}{\pm2}, \frac{\pm6}{\pm2}$

Final List:

$1, -1, 2, -2, 3, -3, 6, -6,$

$\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -\frac{3}{2}$

Example 2: Find all real zeros:

$f(x) = x^3 - 8x^2 + 5x + 14$

constant: (p) = 14 factors: $\pm1, \pm2, \pm7, \pm14$

lead coefficient: (q) = 1 factors: ±1

Possible rational zeros: $\frac{p}{q} = \frac{\pm1}{\pm1}, \frac{\pm2}{\pm1}, \frac{\pm7}{\pm1}, \frac{\pm14}{\pm1}$

$= 1, -1, 2, -2, 7, -7, 14, -14$

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 5 & 14 \\ & \downarrow & 1 & -7 & -2 \\ \hline & 1 & -7 & -2 & 12 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -8 & 5 & 14 \\ & \downarrow & -1 & 9 & -14 \\ \hline & 1 & -9 & 14 & 0 \end{array}$$

-1 is a solution.

$x+1$ is a factor

$f(x) = (x+1)(x^2 - 9x + 14)$

$0 = (x+1)(x-7)(x-2)$

$x+1=0$ $x-7=0$ $x-2=0$

$x=-1$ $x=7$ $x=2$